K21P 4209

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.) Examination, October 2021 (2018 Admission Onwards) MATHEMATICS MAT1C01 : Basic Abstract Algebra

Time : 3 Hours Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries **four** marks.

1. Prove or disprove "the group $\mathbb{Z}_3 \times \mathbb{Z}_3$ is cyclic".
2. Let X be a G-set. Prove that G_x is a subgroup of G for each $x \in X$.
3. Prove that no group of order 30 is simple.
4. Is {(2, 1), (4, 1)} a basis for $\mathbb{Z} \times \mathbb{Z}$? Prove your assertion.
5. Write all polynomials of degree ≤ 3 in $\mathbb{Z}_3[x]$. How many of them are reducible over $\mathbb{Z}_3$?
6. Prove that the p^{th} cyclotomic polynomial is irreducible over $\mathbb{Q}$ for any prime p.

PART – B

Answer **4** questions from this Part without omitting any Unit. **Each** question carries **16** marks.

Unit – I

7. a) Prove that the group $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic and is isomorphic to $\mathbb{Z}_{mn}$ if and only if m and n are relatively prime.

   b) If m is a square free integer then prove that every abelian group of order m is cyclic.

   c) Write all abelian groups of order 32.

P.T.O.

8. a) Let X be a G-set and $x \in X$. Prove that $|Gx| = (G : G_x)$. Also show that if |G| is finite, then $|Gx|$ is a divisor of $|G|$.

b) Let X be a G-set and $Y \subseteq X$ and $G_Y = \{g \in G | gy = y \text{ for all } y \in Y\}$. Show that G_Y is a subgroup of G.

9. a) State and prove First Sylow Theorem.

b) Prove that every group of order p^2, where p is a prime, is abelian.

Unit – II

10. Prove that any integral domain D can be enlarged to a field F such that every element of F can be expressed as a quotient of two elements of D.

11. a) Prove that two subnormal (or normal) series of a group G have isomorphic refinements.

b) Write all composition series of $\mathbb{Z}_{48}$.

12. a) Let $G \neq \{0\}$ be a free abelian group with finite basis. Prove that every bases of G is finite and all basis of G have the same number of elements.

b) Show that $\mathbb{Q}$ under addition is not a free abelian group.

Unit – III

13. a) Let F be a subfield of a field E and α be any element of E. Prove that the map $\phi_\alpha : F[x] \to E$, defined by $\phi_\alpha (a_0 + a_1x + ... + a_nx^n) = a_0 + a_1\alpha + ... + a_n\alpha^n$ is a homomorphism and $\phi_{\alpha|F}$ is the identity map.

b) Prove that every nonzero polynomial $f(x) \in F[x]$ of degree n can have at most n zeros in a field F.

14. a) State and prove Eisenstein Criterion.

b) Let ϕ be a homomorphism of a ring R with unity onto a nonzero ring R′. Let u be a unit in R. Prove that $\phi(u)$ is also a unit in R′.

15. a) Let R be a commutative ring with unity. Prove that M is a maximal ideal of R if and only if R/M is a field.

b) If F is a field, prove that every ideal in F(x) is principal.